Permutations and Combinations

Question1

The number of four digit even number that can be formed using the digits $0, 1, 2$ and 3 without repetition is

KCET 2025

Options:

A. 6

B. 10

C. 4

D. 12

Answer: B

Solution:

Here's a quick breakdown by cases (last digit must be 0 or 2):

Last digit = 0

First digit can be 1, 2 or 3 $\Rightarrow$ 3 choices

The two middle places are filled by the remaining 2 digits in $2! = 2$ ways

$\Rightarrow 3 \times 2 = 6$ numbers

Last digit = 2

First digit can be 1 or 3 (can't be 0 or 2) $\Rightarrow$ 2 choices

The two middle places are filled by the remaining 2 digits in $2! = 2$ ways

$\Rightarrow 2 \times 2 = 4$ numbers

Total = $6 + 4 = 10$.

Answer: Option B (10).

Question2

The number of diagonals that can be drawn in an octagon is

KCET 2025

Options:

A. 15

B. 20

C. 28

D. 30

Answer: B

Solution:

To find the number of diagonals in an octagon (a polygon with 8 sides), use the formula for the number of diagonals in a polygon, which is:

$\frac{n(n-3)}{2}$

where n is the number of sides of the polygon.

For an octagon, $n = 8$. Plug this value into the formula:

$\frac{8(8-3)}{2} = \frac{8\times5}{2} = \frac{40}{2} = 20$

Therefore, an octagon has 20 diagonals.

Question3

Ten chairs are numbered as 1 to 10. Three women and two men wish to occupy one chair each. First the women choose the chairs marked 1 to 6 , then the men choose the chairs from the remaining. The number of possible ways is

KCET 2023

Options:

A. ${}^6P_3 \times {}^4P_2$

B. ${}^6C_3 \times {}^4P_2$

C. ${}^6\mathrm{P}_3 \times {}^4\mathrm{C}_2$

D. ${}^6C_3 \times {}^4C_2$

Answer: A

Solution:

Since the three women choose their chairs first and their choices are limited to the chairs numbered 1 to 6, we must calculate the number of possible ways they can select these chairs. Order matters in this selection because each woman will sit in a unique chair; therefore, permutations should be used.

The number of different ways that the first woman can choose a chair is 6. After she has chosen, there are 5 chairs left for the second woman. The third woman can then choose from the remaining 4 chairs. So, the number of ways the three women can select the chairs is a permutation of 6 items taken 3 at a time, which is denoted by ${}^6P_3$.

After the three women have taken their seats, there are 10 - 3 = 7 chairs left, but the men can only choose from the remaining chairs numbered 7 to 10, so they have 4 chairs to choose from. The number of different ways the first man can choose a chair is 4. After he has chosen, there are 3 chairs left for the second man. So, the number of ways the two men can select from the 4 chairs is a permutation of 4 items taken 2 at a time, which is denoted by ${}^4P_2$.

Now we need to multiply the number of ways the women can sit by the number of ways the men can sit, as these are independent events:

Number of ways women can choose = ${}^6P_3$

Number of ways men can choose = ${}^4P_2$

Total number of ways = ${}^6P_3 \times {}^4P_2$

So the correct answer is Option A:

${}^6P_3 \times {}^4P_2$

Question4

If all permutations of the letters of the word MASK are arranged in the order as in dictionary with or without meaning, which one of the following is 19th word?

KCET 2022

Options:

A. KAMS

B. SAKM

C. AKMS

D. AMSK

Answer: B

Solution:

To find the 19th word in the dictionary order of permutations of the letters in the word "MASK," it is helpful first to determine the total number of permutations for the word. Since "MASK" consists of 4 unique letters, the number of different permutations can be calculated using the factorial of 4:

$4! = 4 \times 3 \times 2 \times 1 = 24$

So, there are 24 possible permutations. Arranging these permutations in dictionary order means starting with the permutations beginning with 'A', followed by those starting with 'K', 'M', and 'S'. Now let's list them out to find the 19th word:

We will list the permutations in blocks based on their starting letter. With each letter in the first position, the remaining three letters can be arranged in 3! or 6 different ways.

Starting with 'A':

- 1. A-K-M-S
- 2. A-K-S-M
- 3. A-M-K-S
- 4. A-M-S-K
- 5. A-S-K-M
- 6. A-S-M-K

Starting with 'K': (Since the dictionary order will have all permutations starting with 'A' first, we continue from the count of 6.)

- 7. K-A-M-S
- 8. K-A-S-M
- 9. K-M-A-S
- 10. K-M-S-A
- 11. K-S-A-M

- 12. K-S-M-A

Starting with 'M':

- 13. M-A-K-S
- 14. M-A-S-K
- 15. M-K-A-S
- 16. M-K-S-A
- 17. M-S-A-K
- 18. M-S-K-A

At this point, we've reached the 18th permutation, which implies the next one will be the 19th. Since we are looking for the 19th word and we have finished with all permutations starting with 'M', we know the 19th word will begin with 'S', as it is the next letter in alphabetical order. The 19th permutation will be:

1. S – A – K – M

Hence, the 19th word formed from the permutation of the letters in "MASK" in dictionary order is "SAKM".

Therefore, the correct answer is:

Option B: SAKM

Question5

A and B are non-singleton sets and $n(A \times B) = 35$. If $B \subset A$, then ${}^{n(A)}C_{n(B)}$ is equal to

KCET 2021

Options:

A. 28

B. 35

C. 42

D. 21

Answer: D

Solution:

Given, $n(A \times B) = 35, B \subset A$
$n(A \times B) = 35$
$n(A \times B) = 7 \times 5$
$n(A \times B) = n(A) \times n(B)$
$\therefore \quad n(A) = 7$ and $n(B) = 5$
${}^{n(A)}C_{n(B)} = {}^{7}C_{5} = {}^{7}C_{2} = 21$

Question6

A student has to answer 10 questions, choosing at least 4 from each of the parts A and B. If there are 6 questions in part A and 7 in part B, then the number of ways can the student choose 10 questions is

KCET 2021

Options:

A. 256

B. 352

C. 266

D. 426

Answer: C

Solution:

Given,

Total number of questions in part $A = 6$

Total number of questions in part $B = 7$

Pattern to choose 10 questions from 13 questions is

4 from part A and 6 from part B or

5 from part A and 5 from part B or

6 from part A and 4 from part B.

$\therefore$ Total required number of ways

$= ({}^6C_4 \times {}^7C_6) + ({}^6C_5 \times {}^7C_5) + ({}^6C_6 \times {}^7C_4)$

$= (15 \times 7) + (6 \times 21) + (1 \times 35)$

$= 105 + 126 + 35 = 266$

Question7

The value of ${}^{16}C_9 + {}^{16}C_{10} - {}^{16}C_6 - {}^{16}C_7$ is

KCET 2020

Options:

A. 0

B. 1

C. ${}^{n}C_0$

D. ${}^{17}C_3$

Answer: A

Solution:

We have,

$${}^{16}C_9 + {}^{16}C_{10} - {}^{16}C_6 - {}^{16}C_7$$
$${}^{16}C_9 + {}^{16}C_{10} - {}^{16}C_{10} - {}^{16}C_9 \, [\because {}^nC_r = {}^nC_{n-r}] = 0$$

Question8

The number of 4 digit numbers without repetition that can be formed using the digits $1, 2, 3, 4, 5, 6, 7$ in which each number has two odd digits and two even digits is

KCET 2019

Options:

A. 450

B. 432

C. 454

D. 436

Answer: B

Solution:

Number of ways of selecting 2 even digits out of 3 digits $= {}^3C_2$

Number of ways of selecting 2 odd digit out of 4 digits $= {}^4C_2$

$\therefore$ required Number of 4 digits number without repetition formed by given digits

$= {}^3C_2 \times {}^4C_2 \times 4! = 3 \times 6 \times 24 = 432$

Question9

If $P(n) : 2^n < n!$ Then the smallest positive integer for which $P(n)$ is true, is

KCET 2019

Options:

A. 4

B. 2

C. 5

D. 3

Answer: A

Solution:

We have, $P(n) > 2^n < n!$

$P(3) : 2^3 < 3!$ which is false

$P(4) : 2^4 < 4!$ which is true

Question10

Everybody in a room shakes hands with everybody else. The total number of handshakes is 45 . The total number of persons in the room is

KCET 2018

Options:

A. 9

B. 10

C. 5

D. 15

Answer: B

Solution:

To determine the total number of people in a room where each person shakes hands with every other person, given that the total number of handshakes is 45, we use the combination formula for choosing 2 people out of n, denoted as $\binom{n}{2}$.

The formula for the number of handshakes is:

$\binom{n}{2} = \frac{n(n-1)}{2}$

We set this expression equal to 45, because the problem states that there are 45 handshakes:

$\frac{n(n-1)}{2} = 45$

Solving the equation:

$n(n-1) = 90$

$n^2 - n - 90 = 0$

Factoring the quadratic equation:

$(n-10)(n+9)=0$

Solving for n, we get:

$n=10 \quad \text{(since n cannot be negative, i.e., } n \neq -9)$

Therefore, the total number of people in the room is 10.

Question11

The number of ways in which 5 girls and 3 boys can be seated in a row so that no two boys are together is

KCET 2018

Options:

A. 14040

B. 14440

C. 14000

D. 14400

Answer: D

Solution:

We have, 5 girl and 3 boys

$\times G \times G \times G \times G \times G \times$

Number of ways no two boys are together is

${}^6P_3 \times 5! = 14400$

Question12

If ${}^nC_{12} = {}^nC_8$, then n is equal to

KCET 2017

Options:

A. 12

B. 20

C. 26

D. 6

Answer: B

Solution:

To solve for n given that ${}^nC_{12} = {}^nC_8$, we can use the property of combinations:

If ${}^nC_x = {}^nC_y$, then one of two conditions must be true:

$x = y$

$x + y = n$

Given the equation ${}^nC_{12} = {}^nC_8$, it's clear that the second condition applies:

$12 + 8 = n$

Therefore, $n = 20$.
